**Errors and uncertainties in physics**

The consideration and appreciation of the significance of the concepts of errors and uncertainties helps to develop skills of inquiry and thinking that are not only relevant to the group 4 sciences. The evaluation of the reliability of the data upon which conclusions can be drawn is at the heart of a wider scientific method, which is explained in section 3 of the “Nature of science” part of the subject guide. Errors and uncertainties are addressed in “Topic 1.2: Uncertainties and error” of the subject guide and this topic can be very effectively treated through the practical scheme of work.

The treatment of errors and uncertainties is also directly relevant in the internal assessment criteria of:

* **Exploration** (“The methodology of the investigation is highly appropriate to address the research question because it takes into consideration all, or nearly all, of the significant factors that may influence the relevance, reliability and sufficiency of the collected data.”)
* **Analysis** (“The report shows evidence of full and appropriate consideration of the impact of measurement uncertainty on the analysis.”)
* **Evaluation** (“Strengths and weaknesses of the investigation, such as limitations of the data and sources of error, are discussed and provide evidence of a clear understanding of the methodological issues involved in establishing the conclusion.”)

**Exploration**

See exemplars that are relevant to addressing errors and uncertainties in the "[Assessed student work](https://ibpublishing.ibo.org/server2/rest/app/tsm.xql?doc=d_4_physi_tsm_1408_1_e&part=8&chapter=1)" section of the TSM.

**Analysis**

The analysis criterion assesses the extent to which the student’s report provides evidence that he or she has selected, recorded, processed and interpreted the data in ways that are relevant to the research question and can support a conclusion.

* The report includes sufficient relevant **quantitative and qualitative raw data** that could support a detailed and valid conclusion to the research question.
* Appropriate and sufficient **data processing** is carried out with the accuracy required to enable a conclusion to the research question to be drawn that is fully consistent with the experimental data.
* The report shows evidence of full and appropriate **consideration of the impact of measurement uncertainty on the analysis**.
* The processed data is correctly interpreted so that a completely valid and detailed conclusion to the research question can be deduced.

**Quantitative and qualitative raw data**

All physics students are expected to deal with uncertainties throughout their investigations.

When numerical data is collected, values cannot be determined exactly, regardless of the nature of the scale or the instrument. If the mass of an object is determined with a digital balance reading to 0.1 g, the actual value lies in a range above and below the reading. This range is the uncertainty of the measurement. If the same object is measured on a balance reading to 0.001 g, the uncertainty is reduced, but it can never be completely eliminated. When recording raw data, estimated uncertainties should be indicated for all measurements.

There are different conventions for recording uncertainties in raw data.

* The simplest convention is the least count, which simply reflects the smallest division of the scale, for example ±0.01 g on a top pan balance.
* The instrument limit of error is usually no greater than the least count and is often a fraction of the least count value. For example, an analogue ammeter is often read to half of the least count division, which would mean that a value of 23 mA becomes 23.0 mA (±0.5 mA). Note that the value is now cited to one extra decimal place so as to be consistent with the uncertainty.
* The estimated uncertainty takes into account the concepts of least count and instrument limit of error but also, where relevant, higher levels of uncertainty as indicated by an instrument manufacturer.

Qualitative and quantitative comments about errors and uncertainties may be relevant in analysis. Qualitative comments might include, but are not limited to, parallax error in reading a scale, reaction time in starting and stopping a timer, random fluctuation in the read-out of a voltmeter, or difficulties in knowing just when a moving ball passes a given point.

Students should do their best to quantify these errors. For example, one student measured a voltage from an unstable power supply and wrote the following qualitative and quantitative comments.

The voltage varied slightly over time; it went up and down by several hundredths of a volt. Therefore, the values recorded have an uncertainty greater than the least significant digit of each measurement. The uncertainty was estimated to be more like ±0.04 V.

Students can make statements about the minimum uncertainty in raw data based on the least significant figure in a measurement, and they can make statements about the manufacturer's claim of accuracy.

If uncertainties are small enough to be ignored, the student should note this fact. In addition, students can make educated guesses about uncertainties depending on the method of measurement.

In physics internal assessment, it is not specified which convention is preferred and a moderator will accept any convention in which the recorded uncertainties are of a sensible and consistent magnitude. It is good practice to write a short statement justifying the chosen uncertainty in each quantity.

The following examples are taken from an experiment to measure the current through and potential difference across a resistor.

**Example 1**

Students need to present raw data in a clear and comprehensible way, including the names of the quantities, the symbols and units, and an estimated raw uncertainty for each raw data quantity (table 1). Uncertainties are always relevant in raw data, even if they are small enough to ignore.

|  |  |
| --- | --- |
| **Voltage *V* / VΔ*V* ≈ 0 V** | **Current *I* / mA Δ*I* = ±0.3 mA** |
| 1.00 | 0.9 |
| 2.00 | 2.1 |
| 3.00 | 2.8 |
| 4.00 | 4.1 |
| 5.00 | 5.0 |
| 6.00 | 5.9 |
| 7.00 | 7.1 |
| 8.00 | 8.0 |
| 9.00 | 8.9 |
| 10.0 | 9.9 |

Table 1

For internal assessment, this could contribute to the attainment of a high level in the analysis criterion.

**Example 2**

In this example (table 2) the uncertainty in the current is too small relative to the precision of the recorded data, although all other aspects are well presented.

|  |  |
| --- | --- |
| **Voltage *V* / VΔ*V* ≈ 0 V** | **Current *I* / mA Δ*I* = ±0.005 mA** |
| 1.00 | 0.9 |
| 2.00 | 2.1 |
| 3.00 | 2.8 |
| 4.00 | 4.1 |
| 5.00 | 5.0 |
| 6.00 | 5.9 |
| 7.00 | 7.1 |
| 8.00 | 8.0 |
| 9.00 | 8.9 |
| 10.0 | 9.9 |

Table 2

For internal assessment, this could contribute to the attainment of a medium level in the analysis criterion.

**Example 3**

In this example (table 3) the student records raw data appropriately in a table, but the symbols are not given, there are no estimated uncertainties and the raw data is recorded with an inconsistent number of significant figures.

|  |  |
| --- | --- |
| **Voltage / V** | **Current / mA** |
| 1 | 0.9 |
| 2 | 2.1 |
| 3 | 2.8 |
| 4 | 4.1 |
| 5 | 5 |
| 6 | 5.9 |
| 7 | 7.1 |
| 8 | 8 |
| 9 | 8.9 |
| 10 | 9.9 |

Table 3

For internal assessment, this could contribute to the attainment of a medium level in the analysis criterion.

**Example 4**

In this example (table 4) the student has not included any units.

|  |  |
| --- | --- |
| **VoltageΔ*V* ≈ 0** | **CurrentΔ*I* = ±0.05** |
| 1.00 | 0.90 |
| 2.00 | 2.10 |
| 3.00 | 2.80 |
| 4.00 | 4.10 |
| 5.00 | 5.00 |
| 6.00 | 5.90 |
| 7.00 | 7.10 |
| 8.00 | 8.00 |
| 9.00 | 8.90 |
| 10.0 | 9.90 |

Table 4

For internal assessment, this could contribute to the attainment of a medium level in the analysis criterion.

**Example 5**

The student may not record any raw data or the presentation and details may be incomprehensible, as in this example (table 5).

|  |
| --- |
| **Raw Data: Voltage and Current** |
| 1 @ 0.9, 2 @ 2.1, 3 @ 2.8, 4 @ 4.1, 5 @ 5, 6 @ 5.9, 7 @ 7.1, 8 @ 8, 9 @ 8.9,10 @ 9.9 |

Table 5

For internal assessment, this could contribute to the attainment of a low level in the analysis criterion.

**Processing data**

**Repeated measurements** allow for calculation of the mean value with associated uncertainty for a quantity. Repeated measurements are used to reduce random errors.

The following examples are taken from an experiment to measure the time it takes for a ball to roll down an inclined plane.

**Example 6**

The student finds the average of three trial measurements of the time it takes for a ball to roll down a 1.00 m inclined plane (table 6). The student clearly and correctly calculates the average time.

|  |  |  |
| --- | --- | --- |
| **Distance *s* / mΔ*s*≈ ±0.01 m** | **Time *t* / sΔ*t* = ±0.01 s** | **Average time** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**/ s**https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/0394.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**= ±0.06 s** |
| 1.00 | 6.286.396.31 | 6.33 |

Table 6







Where represents the mean value and represents the uncertainty in the mean value.

Students may express uncertainties as absolute, fractional, or percentages.

For internal assessment, this could contribute to the attainment of a high level in the analysis criterion.

**Propagating errors**

Random errors in raw data feed through a calculation to give an error in the final calculated result. There is a range of protocols for **propagating errors**. A simple protocol is as follows.

**Note:** A common protocol is that the final total percentage uncertainty should be cited to no more than one significant figure if it is greater than or equal to 2% and to no more than two significant figures if it is less than 2%.

Students should be able to propagate uncertainties through calculations involving addition, subtraction, multiplication, division and raising to a power. They can calculate the uncertainty using the range of data in a repeated measurement. It is good practice to show an example of each type of calculation.

**Error bars**

All students are expected to construct, where relevant, error bars on graphs. In many cases, only one of the two axes will require such error bars. In other cases, uncertainties for both quantities may be too small to construct error bars. A brief comment by the student on why the error bars are not included is then expected. If there is a large amount of data, the student need only draw error bars for the smallest value datum point, the largest value datum point, and several data points between these extremes. Error bars can be expressed as absolute values or percentages.

Arbitrary or made-up error bars will not earn the student credit. Students should be able to use the error bars to discuss, qualitatively, whether or not the plot is linear, and whether or not the two plotted quantities are in direct proportion. In respect of the latter, they should also be able to recognize if a systematic error is present. This is discussed later.

Using the error bars in a graph, students should be able to find the minimum and maximum gradients as well as the minimum and maximum intercepts, and then use these to express the overall uncertainty range in an experiment.

Processed data is usually understood as combining and manipulating raw data to determine the value of a physical quantity. Often raw data is multiplied or divided, added or subtracted from other values or constants. When this is done, errors and uncertainties should be propagated. However, there are cases where the raw data is appropriate for graphing and for establishing a conclusion. For example, in a motion experiment, position and time may be recorded and graphed. In such cases processing will be understood as transferring the data to an appropriate graph, constructing a best-fit line and determining the gradient. The processing of uncertainty consists of correctly constructing the relevant error bars on the graph and correctly determining the gradient and intercept of the graph with uncertainty.

When students process data by product or quotient, sum or difference, or some other mathematical function such as averaging, how well the student processes the raw data determines the mark awarded.

**Example 7**

The student calculates the square of the average time for three trial runs as shown above and also determines the uncertainty.

The average time and uncertainty is:



The uncertainty in average time as a percentage:



The average time squared is:



The uncertainty in time squared is:



The average time squared and its uncertainty is thus:



The datum and its uncertainty are now correctly processed as an error bar on a graph of time squared against distance (figure 7).



Figure 7

For internal assessment, this could contribute to the attainment of a high level in the analysis criterion.

**Example 8**

The student finds the average of three trial measurements of the time it takes for a ball to roll down a 1.00 m inclined plane but expresses the average with too many significant figures (table 7) and does not appreciate the propagation of uncertainty.

|  |  |  |
| --- | --- | --- |
| **Distance *s* / m** | **Time *t* / sΔ*t* = ±0.01 s** | **Average time** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**/ s** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/0394.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**= ±0.01 s** |
| 1.00 | 6.286.396.31  | 6.3266 |

Table 7

The average time and its uncertainty are:



Next the student calculates the square of the average time.

The average time squared is:



The rounding was carried out to be consistent with the uncertainty in the raw data.

Then the student simply carries forward the raw data uncertainty, which is incorrect.



When this is graphed by the student the error bar is insignificant (figure 8), but this is a mistake due to incorrect processing of the uncertainty.



Figure 8

For internal assessment, this could contribute to the attainment of a medium level in the analysis criterion.

**Example 9**

The student could either fail to show any processing of data or process it incorrectly (table 8).

|  |  |  |
| --- | --- | --- |
| **Distance *s* / m** | **Time *t* / s** | **Average time** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**/ s** |
| 1.00 | 6.286.396.31 | 6.32666 |

Table 8

Next the student calculates (but incorrectly records) the square of the average time.

The average time squared is:



There is a major error in the square of the average time and no uncertainties are appreciated.

For internal assessment, this could contribute to the attainment of a low level in the analysis criterion.

The final set of examples is taken from an experiment to determine the acceleration of free-fall by dropping a tennis ball from a range of different heights.

**Example 10**

In an experiment investigating acceleration of free fall of a tennis ball, the student constructs a graph (figure 9) of time squared, *t* 2, against the drop height, *h*, based on the data collected during the experiment (table 9). The student uses the gradient and uncertainties to determine the acceleration of free-fall with respective uncertainty.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Drop Height, *h* / m ±0.02** | **Mean time taken to fall,** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**/ s** | **Uncertainty in the mean time** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/0394.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**/ s** | **% Uncertainty in the mean time** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/0394.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.png**/ %** | **Mean time squared,** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/238/0032.png**/ s²** | **% Uncertainty in the mean time squared** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/0394.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/238/0032.png**/ %** | **Uncertainty in the mean time squared** https://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/0394.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Math/Italic/336/0074.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/336/00AF.pnghttps://xmltwo.ibo.org/publications/mathjax/fonts/HTML-CSS/TeX/png/Main/Regular/238/0032.png**/ s²** |
| 2.00 | 0.43 | 0.03 | 7.3 | 0.18 | 14.7 | 0.03 |
| 3.00 | 0.68 | 0.03 | 4.6 | 0.47 | 9.2 | 0.04 |
| 4.30 | 0.86 | 0.04 | 5.2 | 0.74 | 10.4 | 0.08 |
| 4.80 | 0.93 | 0.03 | 3.4 | 0.86 | 6.8 | 0.06 |
| 6.30 | 1.14 | 0.04 | 3.9 | 1.30 | 7.8 | 0.10 |

Table 9



Figure 9

* Uncertainties can be rounded to one or two significant figures in accordance with the accepted protocol.
* The maximum and minimum lines have been drawn to pass through all of the error bars.

**Gradient of line:** 

**Y-intercept**: 

Starting with and considering the object is dropped from rest, then *u* = 0.





In this experiment the acceleration of free-fall was determined to be .

For internal assessment, this could contribute to the attainment of a high level in the analysis criterion.

**Example 11**

The same data is used, but this time the student has failed to determine maximum and minimum gradients using the uncertainties in time squared (figure 10). As a result he has not been able to determine the range and uncertainty in the calculated value of acceleration of free-fall.



Figure 10

For internal assessment, this could contribute to the attainment of a medium level in the analysis criterion.

**Example 12**

The student has drawn an inappropriate graph with major errors (figure 11). The student has not included error bars or maximum and minimum lines. In addition, the student has drawn a line passing through the points rather than a line of best fit. This has not allowed for the gradient to be determined.



Figure 11

For internal assessment, this could contribute to the attainment of a low level in the analysis criterion.

**Evaluation**

* A detailed conclusion is described and justified which is entirely relevant to the research question and fully supported by the data presented.
* A conclusion is correctly described and justified through relevant comparison to the accepted scientific context.
* Strengths and weaknesses of the investigation, such as limitations of the data and sources of error, are discussed and provide evidence of a clear understanding of the methodological issues involved in establishing the conclusion.
* The student has discussed realistic and relevant suggestions for the improvement and extension of the investigation.

Errors and uncertainties are relevant in evaluation because students are expected to reach a reasonable and justified interpretation of the data and to appreciate the quality of the procedure, making clear reference to the types of error and to the measure of precision and accuracy.

**Random and systematic error**

**Random errors** arise from the imprecision of measurements and can lead to readings being above or below the “true” value. Random errors can be reduced with the use of more precise measuring equipment or its effect minimized through repeat measurements so that the random errors cancel out.

**Systematic errors** arise from a problem in the experimental set-up that results in the measured values always deviating from the “true” value in the same direction, that is, always higher or always lower. Examples of systematic error causes are miscalibration of a measuring device or friction in mechanics experiments. These are typically observed by a non-zero intercept on a graph when a proportional relationship is expected. Making repeat measurements will neither remove nor reduce the systematic error. The direction of any systematic errors should be appreciated.

**Accuracy and precision**

**Accuracy** is how close a measured value is to the expected value, whereas **precision** indicates how many significant figures there are in a measurement. For example, a mercury thermometer could measure the normal boiling temperature of water as 99.5°C (±0.5°C), while a data probe records it as 98.15°C (±0.05°C). In this case the mercury thermometer is more accurate whereas the data probe is more precise.

**Impact of measurement uncertainty on the analysis and interpretation of processed data to deduce a conclusion**

When attempting to measure an already known and accepted value of a physical quantity, such as the charge of an electron or the wavelength of a laser light, students need to appreciate whether or not the accepted value lies within the experimental value range.

* **The error in the measurement can be expressed by comparing the experimental value with the textbook or literature value.**

For example, a student conducts Young’s double-slit experiment and determines that the laser light wavelength is 610 nm. With experimental uncertainty, the student decides that λexp ± Δλexp = (6.1 ± 0.2) × 102 nm. The manufacturer’s literature that came with the laser gives a wavelength of λ = 632.8 nm. The student might write the following.

The accepted value is 6.328 × 102 nm while my experimental value is (6.1 ± 0.2) × 102 nm. The accepted value lies just outside the experimental range, which is from 5.9 × 102 nm to 6.3 × 102 nm. My estimation of errors and uncertainties needs to be re-examined. Nonetheless, my results are close to the accepted value, about 4% too low. This sounds good, but if, in fact, the experimental uncertainty is only 2%, random errors alone cannot explain the difference, and some systematic error(s) must be present.

* **The experimental results fail to meet the accepted value.**

The experimental range does not include the accepted value. The experimental value has an uncertainty of only 2%. A critical student would appreciate that they must have missed something here. There must be more uncertainty and/or errors than acknowledged.

**Example 13**

In example 10 given above, the acceleration of free-fall was determined to be .



Literature value of acceleration of free-fall (IB *Physics data booklet*, (first assessment 2016))



The experimental range of the value of acceleration of free-fall lies between 6.8 and 8.8 m s-2 and this range does not include the literature value.

The fact that % difference > % uncertainty means random errors alone cannot explain the difference and some systematic error(s) must be present. This is also reflected in the fact that the line of best fit has a y-intercept of around 1.3 m.

This shows evidence of a large systematic error as well as large random errors.

Comments regarding a positive vertical shift or a negative horizontal shift in the data points can be discussed as part of the evaluation.

In addition to the above comments, students may also comment on errors in the assumptions of any theory being tested, and errors in the method and equipment being used. Typical examples may include the following.

* A graph of voltage against current does not form a linear and proportional line. It may be that the load resistance is changing as the current changes, so an ohmic relationship does not hold.
* Measuring the magnetic field alongside a current-carrying wire may confirm the inverse relationship, but for the smallest distances and the largest distances the data does not line up. The induction coil has a finite size, and the centre of it is assumed to be zero. This may not be the case. At large distances, the radius is similar in magnitude to the length of the wire, and the inverse law for the magnetic field assumed an infinite wire length.
* When using the motion detector, the software was not calibrated with the speed of sound first, and so the measured distances were inaccurate. This error was due to an unexamined assumption, but it was appreciated when the experimental results were evaluated.
* The experiment was done to determine the efficiency of an electric motor. As the investigation was carried out, the battery may have lost power. This would have affected the results.

Overall, students can critically appreciate limitations in their experimental results due to assumptions in the theory, in the experimental techniques and in the equipment used. Qualitative comments, based on a careful reading of graphed results, will guide students’ criticism.

**Evaluation of procedure and modifications**

See exemplars in the "[Assessed student work](https://ibpublishing.ibo.org/server2/rest/app/tsm.xql?doc=d_4_physi_tsm_1408_1_e&part=8&chapter=1)" section of the TSM.